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Applying Cognitive Conceptual Approach in Developing Year 6 Pupils' Problem Solving Skills in Mathematics

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ABSTRACT

This study aimed to investigate the effect of the cognitive conceptual approach as compared to the heuristics approach in developing pupils' problem solving abilities. An experimental study was conducted with two classes of Year 6 pupils from two different primary schools. Both classes were measured based on the improvement they made on their mathematics achievement tests as well as their responses to a survey. A mathematics achievement test containing ten word problems was designed on the classification systems of Riley, Greeno, and Heller (1983) and Carpenter and Moser (1982). Both classes took the achievement test during the first week of school term as a pre-test and during the eighth week of the school term as a post-test. For the survey, the students were administered four subscales of the Fennema-Sherman Mathematics Attitudes Scale (1976) with the aim to measure changes in their attitudes toward mathematics as a result of the intervention. An independent t-test was run as an indicator for statistical significance at the 0.01 level. Pre-implementation results showed that both classes are comparable in terms of mathematics achievement. For the survey, the control group scored significantly higher than the experimental group in terms of self-confidence, motivation, and enjoyment towards the subject. However, at the post-test, a significant difference was found between the two groups with the experimental group reporting significantly higher scores on the mathematics achievement test as well as the four subscales of the survey.

Keywords: Cognitive acceleration, conceptual knowledge, schema, problem solving

INTRODUCTION

The introduction of 'Thinking Schools Learning Nation' in Singapore in 1997 has placed a greater emphasis on developing thinking among our pupils. As such, to promote greater thinking among Singapore's pupils at the primary school level, there has been increased emphasis on problem solving that are of real life contexts (Gravmeijer, 1994) in the teaching and learning of mathematics.

In Singapore, the teaching of mathematics problem solving in schools is guided by the thematic topics found in textbooks. The word problems typically found at the end of chapters or topics in the prescribed textbooks are primarily designed based on the exercise paradigm (Skovsmose, 2001), where students mainly practice the procedural or algorithmic skills related to the chapter or topic. Riding on the belief that "practice makes perfect", teachers assume that pupils would "learn enough" of these mathematical knowledge by rote as long as they scored well for their examinations even if they have not understood what they have learnt or

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practiced (Adey and Shayer, 2002). A pupil who can only apply an appropriately remembered rule of thumb or heuristic to the solution of a problem without knowing why the rule works only has instrumental understanding (Skemp, 1987) and the acquisition of such knowledge type sometimes lead to a regurgitation of steps in familiar problems.

Therefore, to help pupils become better problem solvers, teachers could help pupils think creatively, reflectively and critically (Richhart, 2002). In fact, teachers should first help their pupils understand what and why they are learning before they can expect them to apply what they have learnt. At the same time, teachers would need to scaffold the problem solving process and address the issues and difficulties faced by pupils, especially in the area of conceptual understanding.

This paper presents a study which aims at providing a curriculum framework for cognitive acceleration to develop pupils' problem solving abilities, particularly in the acquiring of conceptual knowledge. The first part of this paper summarizes the literature review of cognitive acceleration, including an emphasis on schema instruction as well as the core concepts of conceptual knowledge. The second part presents the design and the implementation, while the final part discusses the results of the intervention in the study.

LITERATURE REVIEW

The term cognitive acceleration is based on the theoretical premise that pupils' thinking and ability to learn can be strengthened and developed through systematic training (Demetriou et al., 1992). For the past two decades, there has been a growing development of curriculum design that aimed to accelerate cognitive development in primary school education (Adey and Shayer, 2002). The major goal of these programs was the fostering of pupils' abilities to think effectively and thus increase their general problem solving ability and academic achievement (Adey and Shayer, 1995).

There are three main hypotheses pertaining to cognitive acceleration. The first hypothesis is backed up by several theorists on the central cognitive mechanisms process in the brain (Baddeley, 1990, Pascual-Leone, 1976) whom all believed that there is some kind of general intelligence which operates across all contexts. From the cognitive psychology point of view, since all cognition should be context-dependent (Anderson et al., 1996), educationists should operate on the basis that there is some general cognitive function which can be influenced by the way teachers design their curriculum and this would improve pupils' cognitive abilities. In addition, curriculum materials must also be designed to pitch at a suitable level to challenge cognitive ability of the pupils so that through their active engagement, they can arrive at some form of conclusions together.

The second hypothesis states that a person's cognitive ability develops with age. According to Piaget and Inhelder (1974), a child develops intellectually through the different stages, from sensory-motor stage to formal operational stage. Recognizing these stages and their characteristics can guide teachers to design a curriculum that can develop pupils' cognitive abilities appropriately by leading them from one stage of development to the next.

The third hypothesis states that cognitive development can be influenced by the environment. According to Piaget and Inhelder (1976), cognitive development is seen as a process of balancing and adaptation between how a child sees the world around him and the effects the world has on the child. When the experience the child encounters is coherent with his views, he assimilates in the new experience. But when the experience is contrary to his views, he will change his views to better understand the environment around him. Drawing on these works, the idea is to create an environment that will help stimulate the intellectual mind of our pupils.

Therefore, within the school setting, the development of concrete operational thinking, as characterized by Piaget and Inhelder, can be accelerated in children with a curriculum which provides well-managed cognitive conflict and structured opportunities for social construction, including the encouragement of metacognition.

Based on the above three hypotheses, Adey, Shayer and Yates (1995) conceptualized the six pillars of the cognitive acceleration framework, namely concrete preparation, cognitive conflict, social construction, metacognition, schema theory, and bridging.

Concrete Preparation

Formal operations operate only on a situation that has first been described by the subject in terms of descriptive concrete models. Thus, concrete preparation involves establishing that pupils are familiar with the technical vocabulary, and framework in which a problem situation is set. For example, in getting pupils to do word problems, teachers should go through the language constructs so that pupils are clear what the problems mean.

Cognitive Conflict

Building on the notion of reflective intelligence by Dewey (1963), this term is used to describe an event or observation that the pupil finds puzzling and discordant with his or her previous experience or understanding. Using Piagetian notions of equilibration being attained at a higher level of thinking when a child encounters a problem which cannot be solved with existing cognitive structures, the child goes through a process of either assimilation (they learn a new experience) or accommodation (they change the way they think to fit the new experience). Viewing knowledge itself as problematic, it is therefore not viewed as a fixed body of information, but rather one that is constructed by students themselves.

Social Construction

Drawing on the works of Vygotsky, the construction of knowledge and understanding is a social process. Understanding appears first in the social space, and then becomes internalized by individuals (Vygotsky, 1978). The process of task related oral discussion around new ideas, exploring them through group discussion, asking for explanations and justifications, are all important aspects of pupils' learning (Meloth and Deering, 1999). The notion of a Zone of Proximal Development (ZPD), which is the difference in level between what a child can achieve unaided and what she can achieve with the help of an adult or 'more able peer' also established the fact that learning should be matched in some manner with the child's developmental level. To avoid having the social process ending up as a mere discussion, pupils are trained to respond to others to make the conversation more substantive. Responses may come in either of the following:

a. Questioning: Posing a question to a fellow group member based on what was said; this also includes clarification as well.
b. Proofing: Demonstrating a practical example to substantiate what has been said, e.g. using numerals to show that area of 2 triangles = area of a square.
c. Compare and contrast: Consolidating what has been said by a few individuals and draw out the similar and different points.
d. Extensions: Either building on what someone has said or illustrating with an example to improve the clarity of what has been said.
e. Explanations: To improve the clarity of what was said or to give a reason for a judgment made.

Metacognition

Metacognition is a form of cognition. It involves both knowledge of cognitive processing (how we are thinking) and a conscious control and monitoring of that processing. It is a conscious reflection by a child on his or her own thinking processes. This means that it is a process that must take place *after* a thinking act since at the time when pupils are engaging in a problem-solving activity, their consciousness must be devoted to that. Only then can they think back about the steps they took, and become aware how their own conceptualisation changed during the activity (Perkins and Saloman, 1989).

Effective metacognition during problem solving requires not only knowing what and when to monitor, but also how to monitor. Teaching pupils to be aware of their cognition and better monitoring of their problem solving actions should take place in the context of learning specific mathematics concepts and techniques. However it can be quite challenging as the development of metacognitive skills is difficult and often requires "unlearning" inappropriate metacognitive behaviors developed through previous experiences (Schoenfeld, 1985). For example, in Singapore, pupils are trained to write number sentences. However, midway into the mathematical workings, many pupils tend to forget what their previous numeral workings represent. Therefore, it is important to encourage our pupils to label and explain their workings as they proceed with their number sentences (see *Fig. 1* where labeling is in bold).

Schema Theory

Mathematical problem solving is a transfer challenge requiring children to develop schemas for recognizing novel problems as belonging to familiar problem types for which they know the solutions. According to schema construction theory, a major challenge in effecting mathematical problem solving is the development of schemas for grouping problems into types that require the same solution (Chi et al., 1981). The broader the schema, the greater the probability that individuals will recognize connections between familiar and novel problems and will know when to apply the solution methods they have learned.

Schemas are phrases or words that capture the essence of a concept, event or an experience (Marshall, 1995). They help capture both the pattern of relationships as well as their linkages to operations. Problem sums pose difficulties for many pupils because of the complexity of the solution process (Jonassen, 2003). They emphasises conceptual understanding, knowledge organization and pattern recognition,

Problem 1

In a class of 40 pupils, $\frac{1}{2}$ of the boys is equal to $\frac{3}{4}$ of the girls. How many more boys than girls are the in the class?

1 unit of boys is equivalent to 3 units of girls

$\frac{1}{2}$ of the **boys** is equal to $\frac{3}{4}$ of the **girls**

<u>Method 1:</u>

Making the units equivalent

$\frac{1}{2}$ Boys $\longrightarrow$ $\frac{3}{4}$ Girls

$\frac{3}{6}$ Boys $\longrightarrow$ $\frac{3}{4}$ Girls

10 Units $\longrightarrow$ 40

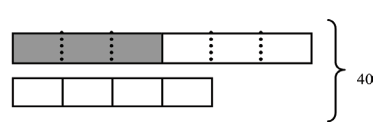
1 unit $\longrightarrow$ 4

2 units $\longrightarrow$ 8

<u>Method 2:</u>

Boys

Girls



10 units $\longrightarrow$ 40

1 unit $\longrightarrow$ 4

2 units $\longrightarrow$ 8

Fig. 1: Example of labeling number sentences in word problems

which are key elements of conceptual knowledge (Jitendra et al., 1999). During problem solving, all problem relevant knowledge is accessible only when the knowledge is adequately organised by a suitable cognitive structure or problem schemata (Chi et al., 1981). This will in turn facilitate problem representation, which translates words into a meaningful representation. According to Skemp (1987), he attributed three functions to the schema: (a) It serves to integrate what is already known. (b) It provides the framework for further learning. (c) It is the basis for understanding.

Coherent with the view of Piaget and Inhelder, Skemp (1987) said that "to understand something is to assimilate it into an appropriate schema" (Marshall, 1995, p.29). Using the word problem in *Fig. 2* as an example, pupils are taught the schema of 'repetition' as in this example where Nancy is repeated. Once they recognise this schema, they can make the units of Nancy the same by using the principle of common multiple and continue to solve the problem.

Problem 2

James had $\frac{3}{4}$ as many sweets as Nancy and Nancy had $\frac{1}{4}$ as many sweets as Tom.

If they have a total of 92 sweets altogether, how many sweets did James have?

Fig. 2: Example of a schema action in word problem

Developing Conceptual Knowledge

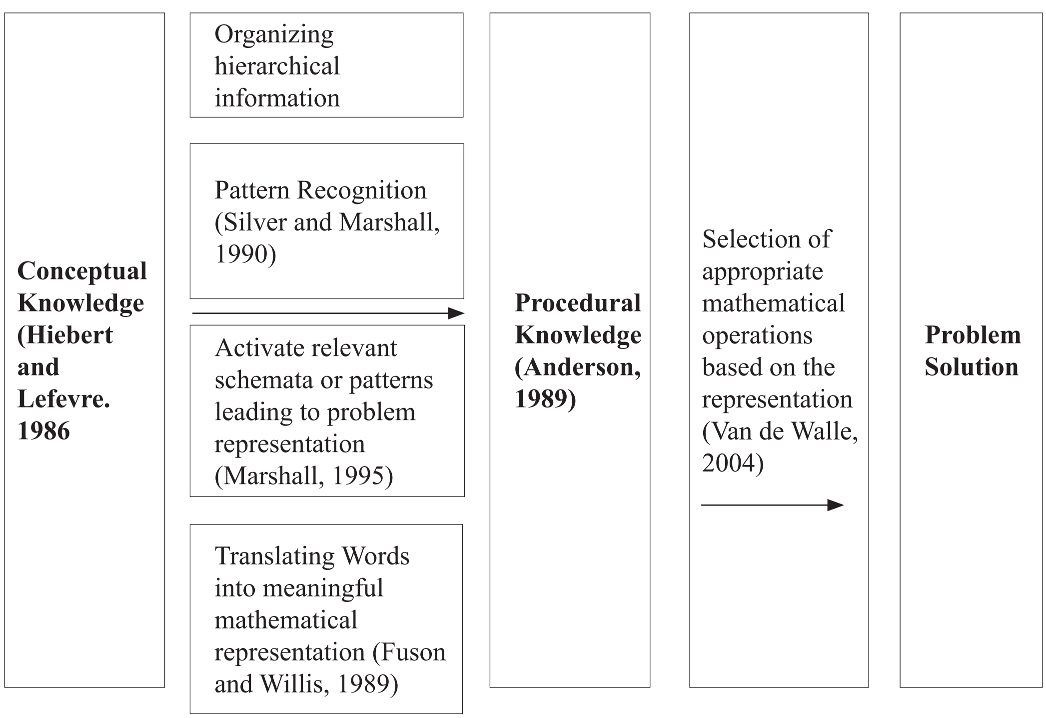


Fig. 3: Components of problem solving

Bridging

The explicit bridging to other contexts is the final link in this chain of developing, abstracting, and generalizing reasoning. Context should be extended to other non-routine problems to let pupils have a trial to see if they can apply what they had learnt in the process across the different topics and settings.

Mathematical problem solving is an extremely complex form of human endeavor that involves much more than the simple recall of facts or the application of well-learned procedures (Lester, 1983). Looking at *Fig. 3*, there are two components that are essential to solving mathematical problems. One is the conceptual knowledge (why and the what) - the facts, concepts and principles, which comprised of individual pieces of information and the relationships between these pieces of information (Hiebert and Lefevre, 1986). The other is the procedural knowledge (how), heuristics in particular, that are used to recall and construct information while solving the problem. This includes both a familiarity with the symbol representation system of mathematics and knowledge of rules and procedures for solving exercises in mathematics.

An important aspect of domain specific concept knowledge is problem comprehension and representation, which involves translating the text of the problem into a semantic representation on the basis of an understanding of the problem structure. While procedural knowledge may or may not be learned meaningfully, conceptual knowledge must be learned with meaning. Procedural knowledge learned without meaning is similar to instrumental understanding, a type of understanding named by Mellin-Olsen (1991) and described by Skemp (1976) as "rules without reason." As such, although procedural knowledge is also important, it is extremely limited unless it is connected to a conceptual knowledge base (Hegarty et al., 1995). Using the word problem in *Fig. 4* as an example:

Without looking at the context, pupils will tend to add $\frac{1}{4} + \frac{2}{3} = \frac{2}{3}$ without realizing that they are related to different bases (orange and apples).

Successful problem solving requires a substantial amount of qualitative and conceptual reasoning (Marshall, 1995). Good problem solvers do not rush to apply a formula or an equation. Instead, they try to understand the problem situation and they consider alternative representations and relations based on the problem statements. Only then are they satisfied that they understand the situation and all problem statements in it in a qualitative way.

PURPOSE OF THE STUDY

Rationale

In some Singapore classrooms, when word problems are presented in the classroom setting as teaching examples, teachers would generally go through the conceptual phase quickly, and almost immediately to the procedural phase by working at the solution (Teong et al., 2004). The conceptual phase appears to be implicit to the teacher, and not often made explicit to the students. Consequently, the emphasis on procedural knowledge accounted for pupils' lack of planning and understanding when they approach a non-routine word problem. They read the questions quickly, do not spend time in understanding and thinking about the concepts, and work towards the solution immediately. Such procedural knowledge learned without meaning is similar to instrumental understanding, a type of understanding described by Skemp (1976) as "rules without reason."

There are a total of 38 apples and oranges in a bag. After $\frac{1}{4}$ of the apples and $\frac{2}{3}$ of the orange are removed, there are 21 apples and oranges left behind. How many oranges are there at first?

Fig. 4: Example of contextual understanding in word problems

In the use of manipulatives as an alternative approach to improving conceptual understanding, teachers often demonstrate the way these manipulatives are to be used and pupils are left with little freedom to give meanings to the experiences in ways that make sense to them; the way the materials are to be used is prescribed. (Cobb et al., 1992). This is based on the belief that mathematics is "out there" and that models "show" the conceptual understanding. However, given the nature of problem solving, learning should be more constructive in nature and pupils should be given greater opportunity to construct their own knowledge.

Building on the previous studies of Hiebert and Lefevre (1986), conceptual knowledge is important to help pupils solve problems successfully. Yet, the underlying challenge is not an issue of what conceptual knowledge is but how teachers can help their pupils construct conceptual knowledge effectively. In looking at successful problem solving, Kantowski (1975) found evidence that although conceptual knowledge is related to success in problem solving, it depended a lot on pupils' cognitive abilities as well. In addition, metacognition has long been linked to successful mathematical problem solving and improvement in the learning of Mathematics (Biggs, 1987; Wittrock, 1986).

SIGNIFICANCE OF THE STUDY

Therefore, this study extends the literature of conceptual knowledge by developing a cognitive conceptual approach; one which makes use of the cognitive acceleration framework to help pupils construct their conceptual knowledge in a constructivist manner. As the intervention would also place a greater emphasis on metacognition, the teacher can also help her pupils to become more aware of their own thinking. At the same time, the study would also examine the attitudes of the pupils towards mathematics and investigate if the new approach could help improve their attitudes towards the subject. If students' attitudes could be improved, research shows this would possibly improve their achievement. As noted by Mager (1968), favorable attitudes toward academic areas will maximize the likelihood that students will remember what they have learned and learn willingly.

RESEARCH PROBLEM

The key research problem is to investigate the effect of the cognitive conceptual approach compared to the heuristics approach to develop pupils' problem solving abilities. To address the research problem, the following specific research questions are:

1. Is there a significant difference in the increase in pupils' mathematics achievement between the cognitive conceptual group and the heuristics group?
2. Is there a significant difference in the increase in pupils' attitude towards mathematics between the cognitive conceptual group and the heuristics group?

In this study, there are two hypotheses tested at 0.01 significance level. The first null hypothesis is that the experimental class would not experience a more significant improvement in their mathematics achievement as compared to the control class. The second null hypothesis is that the experimental class would not experience a more significant gain in their attitude towards Mathematics as compared to the control class.

METHOD

Subjects

Two classes from two government primary schools participated in the study. The control group was an intact class of mixed ability from school A and consisted of 36 pupils. The experiment group is another mixed ability class who comes from school B and consisted of 40 pupils. Both the schools' principals gave permission for their classes to be part of the study.

The two teachers teaching the control and experimental classes were both female and had the same years of teaching experiences. They were of the same age and graduated with a non-Mathematics degree.

Measures

Two types of instruments were used to measure the effect of the cognitive conceptual approach compared to the heuristics approach in the classes, namely mathematics achievement tests and survey questionnaires. The mathematics achievement test, where both pre and post tests are both identical, consist of ten word problems with a maximum score of forty marks. Each of these problems is non-routine in nature and contained multiple numeric and narrative information that is presented in a mixture of mathematical forms (*see* Annex A). Based upon the classification systems of Riley, Greeno, and Heller (1983) and Carpenter and Moser (1982), there are four types of word problems classes. The first type consists of ***change*** (CH) problems which involve an exchange of quantity. The second type consists of ***equalize*** (EQ) problems, which are a variant of the change problems and usually have the phrases "must give away" or "must get". The third problem type consists of the ***combine*** (CB) problems with two specific subtypes. The combine problems require joining and separating sets, but not by any action explicitly indicated in the word problem. The fourth problem type consists of the ***compare*** (CP) problems. To assess the validity and reliability of this test, the items in the mathematics achievement tests have also been sent to Dr Koh Teh Hong[1] and Mr Zuhairi[2] for expert opinion. For this study, an independent t-test is used to measure if the improvement made by the two groups is significant when we measure their progress made from the pre-test to the post-test.

Four of the seven subscales of the Fennema-Sherman Mathematics Attitudes Scales (1976) were used as part of the questionnaire to be administered at the beginning and the end of the eight week intervention (*see* Annex B). The four subscales used in the survey include the following: self confidence, value, motivation, and enjoyment. Each subscale contains four items that were scored on a 5-point Likert scale from 1 (strongly disagree) to 5 (strongly agree). The procedures described by the authors of the instrument were used to analyze the responses: negatively worded items were reverse-scored before analysis so that a 1 represents a strongly agree response and a 5 represents a strongly disagree response. Thus, a high mean on a scale represents a positive attitude toward mathematics. Responses that were left blank were assigned a value of three, a neutral response. The survey was administered by the Head-of-Department (Mathematics) during the first week of the school term (pre-survey) and during the 8th week of the school term (post-survey). All administrations of the surveys were administered during a regularly scheduled class meeting. The duration of the survey was fifteen minutes.

The surveys were administered in their individual classes to minimise location threat and interaction effect as the two schools were very far apart. To validate the questionnaire for use among local students, the survey was also piloted separately at school C to a class of 35 primary six pupils. The Cronbach alpha coefficient (a) was calculated for each of the four dimensions to check for internal consistency. The coefficients are shown below.

According to Kerlinger and Lee (2000), a value of 0.6 or higher is acceptable and indicates the reliability of the scale used. Hence, all the four dimensions are assumed to be sufficiently reliable. Similarly, an independent t-test would be used to measure if there was any significant difference among the control and experiment classes when the study measured their improvement made from the pre-survey to the post-survey based on the four dimensions.

TREATMENT

Pupils were pre-tested the mathematics achievement test that consist of topics such as fractions, ratio, percentage, and whole numbers. The period of intervention was eight weeks and

1

TABLE 1
Internal consistency of Fennema-Sherman Mathematics Attitudes

Dimensions	Cronbach alpha coefficient (a)
Self Confidence	0.883
Value	0.634
Enjoyment	0.852
Motivation	0.823

the number of Mathematics periods was held constant for both the control and experimental groups. The Head-of-Department carried out lesson observation and observation notes were recorded during the eight weeks for scoring fidelity of treatment implementation. Finally, pupils were post tested on the same measures.

Control Group

Within the control group, a prescribed set of problem-solution rules were taught. The teacher approached the problems according to the sequence in which the topics were presented. There was no attempt to broaden pupils' schemas for these problems. Control class instruction provided more practice in applying problem-solution rules and involved a greater emphasis on computational requirements. The mode of instruction was explicit and relied on word examples, guided practice and homework, relying heavily on textbooks and workbooks, as well as the heuristics booklet covering the nine heuristics approach.

Experimental Group

The methodological design of a weekly lesson adopted the six pillars scheme of cognitive acceleration with a dual emphasis on cognitive and conceptual development as shown in *Fig. 5*.

Concrete Preparation and Schema Theory – Laying the Foundation

The teacher began the class by teaching pupils some of the basic algorithmic skills in preparation for the problem formulation. At the same time, the teacher also introduced to pupils the terms of the problem including the context and helped them identify some of the key schemas found in the problem statement. The teacher also took this time to lay the ground rules for the group discussion at the second stage.

Building Conceptual Knowledge through Social Construction

As pupils proceeded in groups to discuss their solutions for the second stage, many would attempt to apply their experiential intelligence (previous experiences and prior knowledge) to solve the problem, but could not. As cognitive dissonance occurred, pupils are engaged in reflective thinking to resolve the cognitive conflict within them. As everyone was involved in clarifying and questioning each other in an attempt to analyze the word problem as a group, they collectively helped build a deeper conceptual understanding of the problem. The groups presented their views again to the class and also responded to questions raised by other group.

Assessing Construction of Ideas within the Group

As they worked within the construction zone, as described by Vygotsky (1978), the teacher would move among the groups and help scaffold the problem to facilitate discussion when necessary. At the same time, she would assess the progress of the pupils' discussion using a set of rubrics (Annex C).

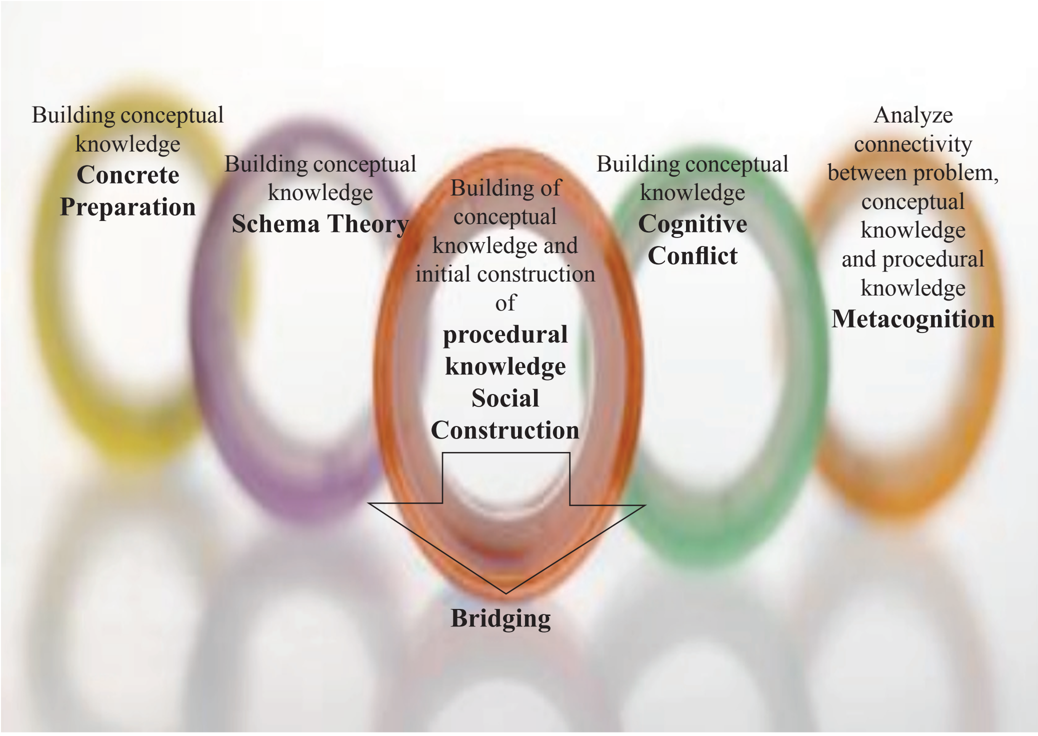


Fig. 5: The Cognitive Conceptual Model

Metacognition

As the pupils entered into the social construction phase, they tended to *monitor* their thoughts more consciously in an attempt to make themselves understood by others within the group. What was often neglected was to help them *evaluate* on what has been done because very often, pupils tend to forget how they had arrived at the answers.

Therefore, instead of giving pupils questions and asking for their solutions, pupils were given problems with worked solutions attached which were incorrect in terms of conceptual representation. As such, pupils needed to evaluate and find out where the mistakes were and offered an alternative solution. This motivated them to adopt a more holistic approach towards analyzing the problem. Please refer to Appendix A for a sample of pupils' responses to the given task.

RESULTS

Analysis of Mathematics Achievement Pre Test

A pre-test was administered to both the control group and the average ability experimental group. Both were comparable as shown by the lack of significant main effects on the pre-test. Table 2 shows the results of the t-test. Thus, in terms of their preparedness for the course, the two groups were considered to be comparable prior the study.

Although both the control and experimental groups improved in their post tests when compared against their pre-tests, the difference gained by the two groups between the pre

TABLE 2
Analysis of pre and post mathematics achievement test

	Control		Experimental		Independent t-test	
Mean	S.D.	Mean	S.D.	Mean	t-value	p-value
Pre-test	8.17	5.15	9.58	4.88	1.22	.225
Post-test	11.49	5.28	35.65	4.89	20.72***	.000
Gain (Post – Pre)	3.32	4.28	26.08	5.76	19.38***	.000

(All figures are round off to 2 decimal places)
p< 0.01***

and post-test was significantly higher for the experimental group compared to the control group (t =19.38, df = 74, p-value = .000 <.05).

ANALYSIS OF PRE QUESTIONNAIRE SURVEY

Self Confidence

Prior to the study, the pre-survey results showed that the difference between the control and experimental group is not significant (t-score = –2.488, df = 74, p-value = .015> .01). Therefore, the control group and the experimental group are comparable in terms of their level of self confidence toward the subject.

Value

In terms of measuring the dimension of 'value' in the survey, the control group and the experimental group are comparable as shown by the lack of significant main effects on the pre-survey. A t-test indicated that the difference between the mean scores of the control group (M = 15.58, S.D. = 2.92) and the experiment group (M = 16.03, S.D. = 3.44) on the pre-survey was not statistically significant (t = 0.600, df = 74, p-value = .55 > .05). Thus, the two groups were considered to be comparable in the way they value the subject of Mathematics.

Enjoyment

With a higher mean score exhibited by the control group in the pre-survey under the dimension of enjoyment, a t-test indicated that the difference between the control and experimental group is significant (t-score = –3.998, df = 74, p-value = .000> .01). Pupils in the control group generally enjoyed the subject of Mathematics more than their peers in the experimental group.

Motivation

Before the intervention, there is no significant difference in the level of motivation towards the subject between the two groups (t-score = –2.315, df = 74, p-value = .023> .01). Therefore, the control group and the experimental group are comparable in terms of their level of self confidence toward the subject.

ANALYSIS OF POST QUESTIONNAIRE SURVEY

Self Confidence

After the intervention, the results showed otherwise. The post survey showed that there is a significant difference in the level of self confidence between the control group and the experimental group (p-value = .001< .01). With a higher mean score (M= 14.09) for the experimental group and a significant difference in the gain between the two groups (t-score = 4.116, df = 74, p-value = .000< .01), pupils in the experimental group actually felt more confident towards the subject than those in the control group. However, the F-test results also showed that this effect might not be solely due

TABLE 3
Analysis of pre and post questionnaire survey

	Control group		Experimental group		Independent t-test	
	Mean	S.D.	Mean	S.D.	t-value	p-value
Self Confidence						
Pre	11.67	2.26	10.15	2.97	–2.49	.015
Post	11.47	3.03	14.05	3.49	3.42***	.001
Gain^	–0.19	3.37	3.90	5.04	4.12***	.000
Value						
Pre	15.58	2.92	16.03	3.44	0.60	.550
Post	14.83	3.94	16.80	3.31	2.36**	.021
Gain^	–0.75	4.79	0.78	4.83	0.23	.172
Enjoyment						
Pre	15.69	2.35	12.98	3.42	–3.99***	.000
Post	14.25	3.49	16.73	3.52	3.07***	.003
Gain^	–1.44	4.29	3.75	4.21	5.33***	.000
Motivation						
Pre	15.17	2.66	13.53	3.43	–2.32	.023
Post	14.86	3.82	16.75	3.38	2.29**	.025
Gain^	–0.31	4.38	3.23	5.14	3.23***	.002

(All figures are round off to 2 decimal places)
^Gain = Post – Pre p< 0.01*** , p< 0.05*

TABLE 4
Test of between-subjects effects

	Group			Treatment		
	Mean sq	F-value	Sig.	Mean sq	F-value	Sig.
Self Confidence	10.67	1.20	0.275	130.09	14.63	0.000
Value	54.95	4.70	0.032	0.01	0.00	0.982
Enjoyment	0.57	0.05	0.817	50.36	4.78	0.030
Motivation	0.58	0.05	0.821	80.75	7.20	0.008

to the intervention alone as the two classes were significantly different at first (F-value = 1.20, significance level at 0.835).

Value

Post- survey results also showed that there is a significant difference in the level of self confidence in the post survey between the control group and the experimental group (t-score = 2.363, df = 74, p–value = .021). However, the difference in gain is not significant when the control group is compared against the experimental group (t-score = 0.225, df = 74, p-value = .172> .05). Although the mean score for the experimental group improved slightly, pupils from both groups believe that Mathematics is important despite the intervention. The real frustration of the pupils is always struggling to understand the subject better and apply what has been taught despite knowing its importance.

Enjoyment

After the intervention, the post survey showed that there is a significant difference in the level of enjoyment between the control group and the experimental group (t-score = 3.071, df = 74, p-value = .003< .01). With a higher mean score (M= 16.73, S.D. = 3.52) for the experimental group and a significant difference in the gain between the two groups (t-score = 5.329, df = 74, p-value = .000< .01), pupils in the experimental group actually enjoyed the subject more than those in the control group. Again, the F-test results also showed that this effect may not be solely due to the intervention alone as there is interaction between the two classes as shown by their significant difference in the beginning. (F-value = 0.05, significance level at 0.817).

Motivation

Under the last dimension of motivation, only the experimental classes experienced an improvement in their motivation towards the subject of Mathematics. Although the difference in the level of motivation at the post survey is only significant at the 95% confidence interval (t-score = 2.288, df = 74, p-value = .025< .05), there is a significant difference in gain in the dimension of enjoyment at the 99% confidence interval when the control group is compared against the average ability experimental (EA) group (t-score = 3.233, df = 74, p-value = .002< .01). Again, the F-test results also showed that this effect may not be solely due to the intervention alone as the two classes are significantly different at first (F-value = 0.05, significance level at 0.821).

DISCUSSION

The present study built on the previous studies of Hiebert and Lefevre (1986) on conceptual knowledge and Adey and Shayer (1995) on cognitive acceleration. By designing a framework (*Fig. 5*) with a dual emphasis to help pupils construct conceptual knowledge through the use of their cognitive abilities, the work was extended. Based on the understanding that schema construction is the key to constructing conceptual knowledge, the study demonstrated how pupils could construct this knowledge in a constructivist manner so that it could be internalised by the pupils (Vygotsky, 1978). In eliciting from her pupils, the experimental class teacher was also consciously improving her questioning techniques so that she can guide them effectively to construct their own knowledge. At the same time, the holistic approach towards analyzing and interpreting the given information also help pupils to consciously monitor their own metacognition, and in turn improved their problem solving abilities (Perkins and Saloman, 1989).

On the basis of the performance in the word problems test, the results supported the hypothesis, with the experimental class's scores significantly higher than the control class (p< 0.000). With the focus on the conceptual understanding of problems, metacognitive activity was displayed more prominently and pupils have begun to understand that the reasoning behind the process was just as important as the final answer. From the interview, the teacher had also observed that there were fewer incidences of pupils playing blindly and more importantly, erroneously, with the given numbers. From the solutions of the pupils in the experimental class, the pupils also appeared to have developed a repertoire of semantic schemes to deal with particular categories of non-routine problem sums. This is very encouraging as the weaker pupils were previously reluctant to attempt challenging problems and would prefer to wait for solutions from teachers. In addition, the teacher also noticed that her pupils in the experimental class had also noticed a reduction in the amount of time taken for them to solve a word problem.

Most of the dimensions in the survey results also supported the hypothesis except on the dimension of how pupils generally valued the subject of Mathematics. This is because pupils from both the control and experimental classes generally viewed Mathematics as an important subject and it is useful to them in their daily lives. It is also very encouraging to know that pupils generally want to do well in the subject, which probably explains why pupils can actually become more confident towards the subject if they can apply effectively what they had learnt and experienced success. According to Mager (1968), favorable attitudes toward school subjects will maximize the likelihood that students will remember what they have learned, willingly learn more about the subject, and use what they have learned.

Contrary to many beliefs about pupil resistance, pupils were generally very open to learning new approaches. The challenge is mainly to make the learning effective for them. In order to help her pupils to develop their conceptual understanding, the teacher mentioned in her interview that she tried to scaffold the problem solving process by not leading the pupils into the procedural and mechanical aspects immediately. In return, her pupils also learnt to appreciate the reasons behind why certain constructs are presented in a particular manner. Through these processes, their self confidence toward the subject naturally increased with greater understanding. The increase in confidence also meant that pupils were generally less anxious towards the learning of mathematics after the intervention period. Although a high level of anxiety can mean that pupils know Mathematics is an important subject and are keen to learn it well, such high level of anxiousness in the long run can be detrimental to learning. Therefore, by helping the pupils to improve in solving word problems, which is a challenge for most primary pupils, pupils will become more confident and improved in their performance. Quite naturally, this would be translated to a higher level of enjoyment towards the subject.

An improvement in pupil motivation towards the subject could also mean that the intrinsic value of learning Mathematics is better appreciated with this holistic and constructivist approach to approaching word problems, as pupils are encouraged to think more actively and critically. The dual emphasis on cognitive and conceptual understanding helped pupils to analyse critically and make sense the underlying meaning of each construct. Because of the success experienced by the pupils in the experimental group, they pupils believe that by trying hard, they can improve in their Mathematics grades. This positive attitude, where pupils are more than willing to put in effort to achieve good grades, gives reasons for the teacher to be optimistic for the future.

CONCLUSION

The major purpose of the present study was to test the hypothesis that the experimental class that is trained using the cognitive conceptual approach would experienced a more significant improvement both in their word problems test and attitude survey as compared to the control class who relies more on the heuristics approach. With respect to theories of cognition and problem solving, the present results are consistent with the assumption that a dual emphasis on cognition and conceptual development can improve pupils' problem solving abilities. Given the relationship between conceptual knowledge for problem representation and solution accuracy, the attempt was to use a constructivist approach to construct such knowledge. Beyond the academic achievement, the significant gain in the self-confidence, enjoyment, and motivation of the pupils toward the subject also supported the fact that knowledge is best constructed by the students themselves when they are given a chance to make meaning with the knowledge imparted (Vygotsky, 1978) although this effect might be due to other factors as well.

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ANNEX A

MATHEMATICS ACHIEVEMENT TEST

Name: ________________ (   )   Marks: ____/40
Class: Primary 6 – (   )   Duration: 1h 30min

Answer all questions. Show your workings clearly.

1. Diane spent $\frac{2}{3}$ of her money on some magazines. She spent 14 of the remaining money on a box of colour pencils.

(a) What fraction of her money was left? (2m)

(b) Given that the box of colour pencils cost $8, find the sum of money Diane had at first. (2m)

2. Ezra had $150 more than Keng Wee. Ezra spent $\frac{3}{4}$ of his money and Keng Wee spent $\frac{4}{7}$ of his money. In the end, Keng Wee and Ezra had the same amount of money left. Find the amount of money Keng Wee had at first. (4m)

3. Mary had $\frac{3}{4}$ as many stickers as Dennis and $\frac{1}{2}$ as many stickers as Roy. If they have a total of 169 stickers altogether, how many more stickers did Roy have than Dennis? (4m)

4. Janet has $\frac{1}{4}$ as many marbles as Kumar. After Kumar gave Janet 8 marbles, Janet now has $\frac{1}{3}$ as many marbles as Kumar. How many marbles did Janet have in the end? (4m)

5. In a class, there are 75% as many boys as girls. After 14 girls and 14 boys left the class, there are now 40% as many boys as girls. How many pupils are there in the class in the end? (4m)

6. Mrs Chen spent $36 on some plates and 75% of her remaining money on some cups. If she had $\frac{1}{6}$ of her total money left, how much money did she have at first? (4m)

7. 30% of Calvin's stickers is equal to 25% of Brian's stickers. If Brian has 24 more stickers than Calvin, what is the total number of stickers Calvin and Brian have? (4m)

8. Ramesh has 50% more cards as Arun and 60% less cards than Jody. If they have a total of 150 cards, how many cards does Jody have? (4m)

9. In a train that was heading for Lido Town, the number of adults was $\frac{4}{5}$ the number of children. Halfway, the train stopped at Clamore Station and 40 children got off the train and another 40 adults had got onto the train. There were then 100% more adults than children. How many adults were there in the train in the end? (4m)

10. In an enrichment class, $\frac{1}{3}$ are boys and the rest are girls. After 3 girls and 3 boys had left the class, there were $\frac{2}{5}$ as many boys as girls remaining in the class. How many pupils are there in the class in the end? (4m)

Classification of Problems

Question	Classification	Question	Classification
1	Combine	6	Combine
2	Equalize	7	Equalize
3	Compare	8	Compare
4	Change	9	Change
5	Change	10	Change

Based on Riley, Greeno and Heller (1983) and Carpenter and Moser (1982)

ANNEX B

Mathematics Attitudes Survey

Name: ____________________ (   )   Class: Primary 6 – (   )
School: ______________

Read each question below carefully. Circle 1, 2, 3, 4 or 5 to indicate the extent you agree with the statement by using the following scale:

1. **Strongly Disagree**
2. **Disagree**
3. **Neutral**
4. **Agree**
5. **Strongly Agree**

1.	Doing mathematics problem sums make me feel nervous	1 2 3 4 5
2.	Mathematics is important in everyday life	1 2 3 4 5
3.	I enjoy studying math in school	1 2 3 4 5
4.	I am willing to learn more than the required amount of mathematics	1 2 3 4 5
5.	I am able to solve Mathematics problems without too much difficulty	1 2 3 4 5
6.	Doing well in Mathematics is not important for my future	1 2 3 4 5
7.	Mathematics is dull and boring	1 2 3 4 5
8.	I would like to avoid using mathematics in in my future studies	1 2 3 4 5
9.	I think I can handle difficult Mathematics problems	1 2 3 4 5
10.	I study Mathematics because I know how useful it is	1 2 3 4 5
11.	I am happier in a Mathematics class than in any other class	1 2 3 4 5
12.	I plan to take as much Mathematics as I can during my education	1 2 3 4 5
13.	I can get good grades in Mathematics	1 2 3 4 5
14.	I don't expect to use much Mathematics when I get out of school	1 2 3 4 5
15.	Taking Mathematics is a waste of time	1 2 3 4 5
16.	I see mathematics as something I won't use very often when I get out of high school	1 2 3 4 5

Adapted from Fennema, E. and Sherman, J.A. (1976). Fennema – Sherman Mathematics Attitudes Scales: Instrument designed to ensure attitudes towards the learning of mathematics by males and females. *JSAS Catalog of Selected Documents in Psychology*, *6*(1), 3b.

ANNEX C

Five categories of thinking behaviors checklist

	Category	Characteristics	Tick
A	Explanation	A child explains: - his idea/action - another child's idea/action - his/her idea of explanation -his/another child's misunderstanding/difficulty	
B	Compare & Contrast	A child either: - summarizes what was said by a few individuals - draws out coherent views - highlights main ideas which are different	
C	Proofing	A child uses additional data to back up a statement that was made.	
D	Extensions	A child makes an extension when he: - makes various suggestions about solving a problem - Builds on each other's idea or use several sources of information to solve a problem; - highlights main ideas which are different - agrees that a problem is not solvable and give reason/s to back it up.	
F	Questioning	A child asks questions to the teacher or another child to clarify task/ activity/ problem/ ideas.	

APPENDIX A

Excerpts of pupils' response from metacognition assessment

Students' Responses to question 2

Student A: Ron should have 125% because Ben is 100%. Ben was not repeated twice and Joe should be 100%. The repeated name units must be the same. The repeated name should be 60% because Joe have 100%"

Student B: Ron is 25% more than Ben. Ron is repeated twice. Ben must be 100%. The repeated name must have the same units. 'And' refer to Ron, not Ben. Ron is 40% lesser than Joe. Joe must be 100%. How to make Ron have the same units? Use the lowest common multiply of the units that have at first."

Student C: Lesser needs to '-'minus and after than is a person that is have 100%. Ron is comparing with Ben so Ron is 125% because more means + and Ben is 100% because he is the one that Ben is comparing with. comparing --> means that person 100%.

Student D: 100%, Ron has 25% more so 100% + 25% = Ron's cards.

Student E: You should have put 125% for Ron as he has 25% more than Ben. Do not minus off 25% from 100% unless the qn says that Ron has 25% lesser than Ben. But now the qn says that Ron has 25% more than Ben so you should add 25% to 100% hence it is all wrong.

Student F: Ron should be 125% as it is 25% more than Ben so Ben is 100%. Joe is 100%, Ron is 40% lesser than Joe so Ron is 60%.

Student G: First of all, the qn said that Ron has 25% more cards than Ben. Secondly, and 40% lesser cards than Joe means that Ron has 40% lesser cards than Joe. Actually, Ron is a repeated identity not Ben.